Higgs sector and *R*-parity breaking couplings in models with broken $U(1)_{B-L}$ gauge symmetry

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Four different supersymmetric models based on $SU(2)_L \times U(1)_R \times U(1)_{B-L}$ and $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ gauge symmetry groups are studied. $U(1)_{B-L}$ symmetry is broken spontaneously by a vacuum expectation value (VEV) of a sneutrino field. The right-handed gauge bosons may obtain their mass solely by a sneutrino VEV. The physical charged lepton and neutrino are mixtures of gauginos, Higgsinos and lepton interaction eigenstates. Explicit formulas for masses and mixings in the physical lepton fields are found. The spontaneous symmetry-breaking mechanism fixes the trilinear *R*-parity breaking couplings. Only some special *R*-parity breaking trilinear couplings are allowed. There is a potentially large trilinear lepton number breaking coupling — which is unique to left-right models — that is proportional to the $SU(2)_R$ gauge coupling g_R. The couplings are parametrized by few mixing angles, making spontaneous *R*-parity breaking a natural ''unification framework'' for *R*-parity breaking couplings in supersymmetric left-right models.

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I. INTRODUCTION

A major problem in supersymmetry is related to the lepton and baryon numbers, which seem to be conserved to a very high precision. In the standard model (SM) lepton- or baryon number violating renormalizable interactions do not exist due to the particle content and gauge symmetry. In the minimal supersymmetric standard model (MSSM), instead, given all the supersymmetric partners of standard model particles, one would expect *a priori* both lepton and baryon numbers to be violated. On the baryon and lepton number-violating couplings there are, however, strong experimental constraints. The most notable of the limits follows from the nonobservation of nucleon decay, which sets extremely stringent limits on the products of lepton and baryon number-violating couplings [1].

One can cure the problem by assuming that so-called *R* parity is conserved. *R* parity is defined by $R = (-1)^{3(B-L)+2S}$ where B and L are the baryon and lepton numbers of respective fields and S is spin. If *R* parity is conserved, the proton is stable. Also the lightest supersymmetric particle (LSP), which usually is a neutralino, does not decay and is thus a good candidate for dark matter. Because of conserved *R* parity the supersymmetric particles can be only produced in pairs in collider experiments [2]. Conservation of *R* parity is a much stronger assumption than is phenomenologically necessary. It suffices that either baryon or lepton number violating interactions are strongly suppressed to avoid proton decay, and that the remaining interactions are small enough not to have been directly observed.

If the *R* parity were a gauge symmetry, it would be protected against violations arising, for example, from quantum gravity. Attractive alternative to a global symmetry would thus be a local *R* parity. This can be realized in a theory based on a gauge group that has $B-L$ symmetry as a discrete subgroup. An interesting low-energy theory with this property is the supersymmetric left-right (SUSYLR) theory obeying the gauge symmetry $SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$, where the *R* parity is a discrete subgroup of $U(1)_{B-L}$ gauge symmetry. This model can be embedded in a supersymmetric $SO(10)$ theory [3,4].

It is possible that in the process of spontaneous symmetry breaking this kind of model developes a minimum that violates *R* parity. As there are no neutral fields carrying baryon number, it will always remain unviolated. Electrically neutral sneutrinos, however, carry lepton number, so that a nonvanishing vacuum expectation value (VEV) of a sneutrino would lead to lepton number violation and breaking of the *R* parity [5]. In some versions of SUSYLR model a nonvanishing sneutrino VEV is in fact unavoidable [6,7]. The *R*-parity violating interactions are then determined by the spontaneous symmetry-breaking mechanism.

Much work on *R*-parity breaking by sneutrino VEV has been done in the framework of MSSM with explicit *R*-parity breaking terms [8]. One of the main differences between *R*-parity breaking MSSM and SUSYLR models are that physical spontaneous symmetry breaking is very nontrivial in SUSYLR models, strongly restricting the parameters of the model. SUSYLR model has more gaugino and Higgsino fields than MSSM, and as a result, there is a set of *R*-parity violating Yukawa interactions that are unique to the SUSYLR models. Left-right models give also a solution to the neutrino mass puzzle: the neutrino mass is naturally small due to so-called seesaw mechanism. If the *R* parity is broken spontaneously the nature of the seesaw mechanism that gives the neutrino mass changes, as the neutrino is mixed with Higgsinos.

In [7] the Higgs sector of the left-right models with spontaneous *R*-parity violation was studied in detail. In this work I will study the mass spectrum and couplings of the Higgs fields more in detail. I will also investigate the fermion sector and the *R*-parity breaking couplings in this class of models.

In this work a bottom-up approach will be used: first I define four phenomenologically viable models having gauged $B-L$ symmetry. I discuss the Higgs sector of these models. The *R*-parity breaking manifests itself in the fact that some scalar and fermion mass eigenstates are mixtures of fields with different *R*-quantum numbers. I give mass formulas and compositions for physical charged and neutral lepton fields in terms of the model parameters, and analyze their interactions with Higgs fields and gauge bosons. A summary of resulting *R*-parity breaking Yukawa interactions is given. In order to handle large fermion mass matrices we need to use some approximative methods, which are described in the Appendixes.

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II. DESCRIPTION OF MODELS

The minimal left-right models involving gauged $U(1)_{B-L}$ symmetry can be divided into two classes: either the right-handed symmetry breaking is accomplished by the VEV of the right-handed sneutrino VEV (models 1a and 1b), in which case the right-handed scale is limited to the TeV range; or there are $SU(2)_R$ triplet fields that contribute to the symmetry breaking (models 2a and 2b). In the latter case the right-handed scale, and thus the mass of the extra gauge bosons, can be arbitrarily heavy.

By minimal we mean that the models have minimal phenomenologically acceptable supersymmetric particle content for a chosen gauge symmetry group and for a chosen scale of vacuum expectation values. We do not, however, set any *a priori* constraints on the couplings of the model. In the following, we list the particle content of four such models. The spontaneous R-parity violation is unavoidable in three of these models: in models 1a and 1b a nonvanishing sneutrino VEV is needed to give phenomenologically acceptable masses (~ 1 TeV) to the right-handed gauge bosons. In model 2b the R parity must also be spontaneuously broken, unless the model is expanded with nonrenormalizable interaction terms or extra Higgs fields [6,7]. Model 2a has both R-parity violating and conserving physical vacuum solutions. In this work we, however, concentrate solely on the R-parity violating solutions.

A. Model 1a: $U(1)_R$ and $v_R \sim 1$ TeV

The minimal SUSYLR model obeying gauge symmetry $SU(2)_L \times U(1)_R \times U(1)_{B-L} \times SU(3)_C$ has the same chiral superfields as the MSSM, except that there are additional right-handed neutrino superfields. The superfield content of the model is thus the following ($i=1,2,3$):

$$L_L^i = \begin{pmatrix} \nu_L^i \\ e_L^i \end{pmatrix} (\mathbf{2},0,-\tfrac{1}{2},\mathbf{1}),$$

$$e_R^i(\mathbf{1},\tfrac{1}{2},\tfrac{1}{2},\mathbf{1}),$$

$$\nu_R^i(\mathbf{1},-\tfrac{1}{2},\tfrac{1}{2},\mathbf{1}),$$

$$Q_L^i = \begin{pmatrix} u_L^i \\ d_L^i \end{pmatrix} (\mathbf{2},0,-\tfrac{1}{6},\mathbf{3}),$$

$$d_R^i(\mathbf{1},\tfrac{1}{2},\tfrac{1}{6},\mathbf{3^*}),$$

$$u_R^i(\mathbf{1},-\tfrac{1}{2},\tfrac{1}{6},\mathbf{3^*}),$$

$$\phi_1 = \begin{pmatrix} \phi_{11}^0 \\ \phi_{11}^- \end{pmatrix} (\mathbf{2},-\tfrac{1}{2},0,\mathbf{1}),$$

$$\phi_2 = \begin{pmatrix} \phi_{22}^+ \\ \phi_{22}^0 \end{pmatrix} (\mathbf{2},\tfrac{1}{2},0,\mathbf{1}). \qquad (1)$$

The most general renormalizable superpotential for these fields can be written as

$$W_{1a} = \lambda_\nu \phi_2^T i\tau_2 L_L \nu_R + \lambda_e \phi_1^T i\tau_2 L_L e_R + \lambda_u \phi_2^T i\tau_2 Q_L u_R + \lambda_d \phi_1^T i\tau_2 Q_L d_R + \mu_\phi \phi_1^T i\tau_2 \phi_2, \qquad (2)$$

where generation indices have been suppressed. The resulting scalar potential is minimized by the following set of VEV's:

$$\langle \tilde{\nu}_R \rangle = \sigma_R \simeq v_R, \quad \langle \phi_{11}^0 \rangle = v_d, \quad \langle \phi_{22}^0 \rangle = v_u, \quad \langle \tilde{\nu}_{Lk} \rangle = \sigma_{Lk}. \qquad (3)$$

The symmetry breaking proceeds at two stages: at scale v_R $U(1)_R \times U(1)_{B-L}$ is broken by sneutrino VEV σ_R to the hypercharge symmetry $U(1)_Y$ of the standard model. The residual $SU(2)_L \times U(1)_Y$ symmetry is further broken to $U(1)_{em}$ at the weak scale. The gauge couplings of respective symmetry groups obey relation $g_Y^{-2}(v_R) = g_R^{-2}(v_R) + g_{B-L}^{-2}(v_R)$.

The σ_R appears in D terms in squark mass-squared matrices. The VEV σ_R is at most of the order of the soft supersymmetry breaking mass squared terms of the SM quarks ($\tilde{m}_{Q_L}^2$ and $m_{Q_R}^2$), if the $U(1)_{em} \times SU(3)_C$ gauge symmetry is to remain unbroken [9]:

$$\frac{1}{8} g_R^2 |D| \lesssim \frac{1}{2} (\tilde{m}_{Q_L}^2 + \tilde{m}_{Q_R}^2) \sim (1 \text{ TeV})^2, \qquad (4)$$

where $\tilde{m}_{Q_L}^2$ and $\tilde{m}_{Q_R}^2$ are the soft mass squared terms for the squarks and where the D term is in model 1a

$$D \equiv \sigma_R^2. \qquad (5)$$

The sneutrino VEV $\langle \tilde{\nu}_R \rangle = \sigma_R$ contributes, along with the VEV's of the Higgs doublets, to the mass of the right-handed gauge bosons W_R and Z_R:

$$m_{W_R}^2 = \frac{1}{2} g_R^2 (\sigma_R^2 + v_d^2 + v_u^2),$$

$$m_{Z_R}^2 \simeq \frac{1}{2} (g_R^2 + g_{B-L}^2)(\sigma_R^2 + v_d^2 + v_u^2). \qquad (6)$$

The VEV of the left-handed sneutrino $\langle \tilde{\nu}_L \rangle = \sigma_L$ contributes to the mass of the W_L boson, which is given by

$$m_{W_L}^2 = \frac{1}{2} g_L^2 (v_u^2 + v_d^2 + \sigma_L^2). \qquad (7)$$

The physical top quark mass is related to the Yukawa coupling λ_t by the modified minimal subtraction scheme $(\overline{\text{MS}})$ relation $m_t/(1+4\alpha_s/3\pi) = \lambda_t v_u$. If the mass of the top quark m_t is taken to be $m_t = 175$ GeV, the requirement that the Yukawa coupling λ_t is in perturbative region ($\lambda_t^2 < 4\pi$) yields the limit $\sigma_L \lesssim 168$ GeV. This limit could be further improved to by requiring that the top Yukawa coupling remains perturbative up to some higher scale. Requiring perturbativity up to the grand unified theory scale $\sim 2 \times 10^{16}$ GeV sets the limit to about $\sigma_L \lesssim 90$ GeV.

B. Model 1b: $SU(2)_R$ and $v_R \sim 1$ TeV

In order to make the parity symmetry explicit the right-handed gauge group can be promoted from $U(1)_R$ of model 1a to $SU(2)_R$. Explicit parity symmetry thus motivates one to extend the gauge group to $SU(2)_L \times SU(2)_R \times U(1)_{B-L} \times SU(3)_C$. As in model 1a, the left-right symmetry group is broken to the MSSM symmetry group at scale $v_R \gtrsim 1$ TeV.

The chiral superfields of the minimal version of the model are $(i=1,2,3)$:

$$L_L^i = \begin{pmatrix} \nu_L^i \\ e_L^i \end{pmatrix} \quad (\mathbf{2},\mathbf{1},-\tfrac{1}{2},\mathbf{1}),$$

$$L_R^i = \begin{pmatrix} e_R^i \\ \nu_R^i \end{pmatrix} \quad (\mathbf{1},\mathbf{2},\tfrac{1}{2},\mathbf{1}),$$

$$Q_L^i = \begin{pmatrix} u_L^i \\ d_L^i \end{pmatrix} \quad (\mathbf{2},\mathbf{1},-\tfrac{1}{6},\mathbf{3}),$$

$$Q_R^i = \begin{pmatrix} d_R^i \\ u_R^i \end{pmatrix} \quad (\mathbf{1},\mathbf{2},\tfrac{1}{6},\mathbf{3^*}),$$

$$\phi_k = \begin{pmatrix} \phi_{1k}^0 & \phi_{2k}^+ \\ \phi_{1k}^- & \phi_{2k}^0 \end{pmatrix} \quad (\mathbf{2},\mathbf{2},0,\mathbf{1}) \quad (k=1,2). \tag{8}$$

The fields obtain VEV's as in model 1a, Eq. (3). Equations (4) and (5) for the D term as well as Eqs. (6) and (7) for gauge boson masses are valid also in this case. As in model 1a, the sneutrino VEV $\langle \tilde{\nu}_R \rangle = \sigma_R$ contributes to the masses of right-handed gauge bosons. Note that as long as the right-handed scale v_R is close to the supersymmetry breaking scale M_{SUSY}, as defined by Eq. (4), the $SU(2)_R$ triplet fields are not needed for symmetry breaking. The superpotential can be written as

$$\begin{aligned} W_{1b} = & L_L^T i\tau_2(\lambda_\nu \phi_2 + \lambda_e \phi_1) L_R \\ & + Q_L^T i\tau_2(\lambda_d \phi_2 + \lambda_u \phi_1) Q_R \\ & + \sum_{i,j=1}^{2} \mu_\phi^{ij} \,\mathrm{Tr}\, \phi_i i\tau_2 \phi_j^T i\tau_2, \end{aligned} \tag{9}$$

where lepton and quark family indices have been suppressed.

We have checked that there is a realistic radiative symmetry breaking by explicitly calculating the full physical scalar spectrum.

C. Model 2a: $U(1)_R$ and $v_R \gg 1$ TeV

In order to have a physical symmetry breaking the D terms can be at most of the supersymmetry breaking scale. All squarks would not have physical masses, if the D term related to $U(1)_R$ and $U(1)_{B-L}$ gauge symmetries would be large [see Eq. (4)]. In order to facilitate the right-handed symmetry breaking at some large scale $v_R \gg M_{SUSY}$ one must add fields that cancel out the large contributions to the D terms.

The minimal anomaly-free addition to model 1a that cancels the large contributions to the D term related to *both* the $U(1)_R$ *and* $U(1)_{B-L}$ gauge symmetries is a pair of δ fields:

$$\begin{aligned} &\delta_R(\mathbf{1},-1,1,\mathbf{1}), \\ &\Delta_R(\mathbf{1},1,-1,\mathbf{1}). \end{aligned} \tag{10}$$

The most general gauge-invariant renormalizable superpotential is

$$W_{2a} = W_{1a} + f_R \nu_R \nu_R \Delta_R + \mu_{\Delta R} \delta_R \Delta_R. \tag{11}$$

These fields will obtain VEV's $\langle \delta_R \rangle = v_{\delta_R}$ and $\langle \Delta_R \rangle = v_{\Delta_R}$. The D term related to $U(1)_R$ and $U(1)_{B-L}$ gauge groups is then

$$|D| \equiv |\sigma_R^2 + 2v_{\delta_R}^2 - 2v_{\Delta_R}^2| \lesssim M_{SUSY}^2. \tag{12}$$

Model 2a has been studied extensively in the case of conserved R parity in [10].

D. Model 2b: $SU(2)_R$ and $v_R \gg 1$ TeV

In this case, as in model 2a, the D terms can be at most of the order of the supersymmetry breaking scale. As before, in order to cancel the contributions both to the D term related to the both $SU(2)_R$ and $U(1)_{B-L}$ gauge symmetries one must introduce extra fields in addition to those appearing in model 1b. The minimal addition is a pair of triplet superfields:

$$\Delta_R = \begin{pmatrix} \frac{1}{\sqrt{2}}\Delta_R^- & \Delta_R^0 \\ \Delta_R^{--} & -\frac{1}{\sqrt{2}}\Delta_R^- \end{pmatrix} \quad (\mathbf{1},\mathbf{3},-1,\mathbf{1}),$$

$$\delta_R = \begin{pmatrix} \frac{1}{\sqrt{2}}\delta_R^+ & \delta_R^{++} \\ \delta_R^0 & -\frac{1}{\sqrt{2}}\delta_R^+ \end{pmatrix} \quad (\mathbf{1},\mathbf{3},1,\mathbf{1}). \tag{13}$$

Model 2b has been studied in [4,7,9,11,12]. In the minimum of the scalar potential these fields acquire nonvanishing VEV's $\langle \Delta_R^0 \rangle = v_{\Delta_R}$ and $\langle \delta_R^0 \rangle = v_{\delta_R}$. One can, in order to preserve explicit left-right symmetry, add corresponding $SU(2)_L$ triplet fields Δ_L and δ_L. With suitable choice of parameters they decouple from the scalar and fermion mass matrices. Therefore, for simplicity, they will not be taken into account in the following discussion.

The superpotential of the model is

$$W_{2b} = W_{1b} + f_R L_R^T i\tau_2 \Delta_R L_R + \mu_{\Delta R} \mathrm{Tr}\, \Delta_R \delta_R. \tag{14}$$

The spontaneous R-parity breaking is unavoidable in this model [6], the sneutrino having necessarily a nonvanishing VEV, $\langle \tilde{\nu}_R \rangle = \sigma_R \neq 0$, in all minima of the scalar potential that conserve the electric charge. One could, however, modify the model in such a way that the sneutrino VEV vanishes and there is no R-parity violation. This could be done, for example, by adding one $SU(2)_R$ triplet that is singlet under $U(1)_{B-L}$ or by introducing some nonrenormalizable operators to the superpotential [7,13].

In Appendix A we have found a global minimum for the models 2a and 2b. At the limit of the large right-handed scale v_R the right-handed VEV's v_{δ_R}, v_{Δ_R}, and σ_R are typically of the same order

$$\sigma_R \sim v_{\delta_R} \sim v_{\Delta_R} \sim v_R, \tag{15}$$

while the D term (12) is of the order of the supersymmetry breaking scale $M_{SUSY}^2 \sim (1\ \text{TeV})^2 \ll v_R^2$. In particular, it would be natural to have the sneutrino VEV σ_R of the order of the right-handed scale v_R.

The large VEV σ_R takes the model away from the supersymmetric minimum of the scalar potential. This could result in the need for fine tuning in model parameters. Fine tuning is not needed, however, if the couplings obey the following relations (see Appendix B):

$$|\mu_\phi| \lesssim M_{SUSY}, \quad |\mu_{\Delta R}| \lesssim M_{SUSY},$$
$$|\lambda_\nu \sigma_R| \lesssim M_{SUSY}, \ \text{and} \ |f_R v_R| \lesssim M_{SUSY}. \tag{16}$$

III. HIGGS SPECTRUM

In all models, the scalar sector is larger than that of the MSSM. The requirement that the minimum of the scalar potential conserves electric charge and color (i.e., all scalar mass-squared eigenvalues are non-negative) restricts the parameter space. A numerical example of full Higgs spectrum of model 2b is given in Appendix C.

A. Light neutral Higgs scalar

The light Higgs spectrum is characterized by one light neutral Higgs scalar

$$h \simeq \cos\beta\, \mathrm{Re}(\phi_{11}^0) + \sin\beta\, \mathrm{Re}(\phi_{22}^0), \tag{17}$$

where $\tan\beta = v_u/v_d$ is the ratio of Higgs bidoublet VEV's. It has a tree-level upper limit for its mass [14,7]:

$$m_h^2 \leq \left(1 + \frac{g_R^2}{g_L^2}\right) m_{W_L}^2 \cos^2 2\beta. \tag{18}$$

The radiative corrections to the limit (18) have been calculated in [7], and it was found that they increase the tree-level upper bound on the mass of m_h typically by ~ 30 GeV.

The limit (18) can be made stricter by taking the heavy ($\sim m_{Z_R}$) Higgs direction into account. The 2×2 submatrix M^2 of the full mass matrix of model 2b is

$$M_{11}^2 = \frac{1}{2}(g_L^2 + g_R^2) M_L^2 \cos^2 2\beta,$$

$$M_{12}^2 = M_{21}^2 = -2\lambda_\nu^2 M_L M_R \sin^2\beta x^2 + \frac{1}{2} g_R^2 M_L M_R (-\cos 2\beta),$$

$$M_{22}^2 = \frac{1}{2}(g_R^2 + g_{B-L}^2) M_R^2 + 24 f_R \mu_{\Delta R} v_{\delta R} x^2 - 12 f_R A_R v_{\Delta R} x^2 - \frac{16 \mu_{\Delta R} B_{\Delta R} v_{\Delta R} v_{\delta R}}{M_R^2} + 4 f_R^2 x^2 (\sigma_R^2 - 8 v_{\Delta R}^2), \tag{19}$$

where the scalar fields are taken to the light direction (17) and to the direction $1/N[2v_{\Delta_R}\mathrm{Re}(\Delta_R^0) - 2v_{\delta_R}\mathrm{Re}(\delta_R^0) - \sigma_R \mathrm{Re}(\tilde{\nu}_R)]$ corresponding to the heavy Higgs, which we will discuss later in this section. We have used $M_L^2 = v_u^2 + v_d^2 = 2m_{W_L}^2/g_L^2$, $M_R^2 = \sigma_R^2 + 4v_{\Delta R}^2 + 4v_{\delta R}^2 = 2m_{Z_R}^2/(g_R^2 + g_{B-L}^2)$ and $x = \sigma_R/M_R$. The limit (18) can be saturated only if the nondiagonal element M_{12}^2 is small, that is, the product of neutrino Yukawa coupling λ_ν and R-parity breaking parameter x is $\lambda_\nu x \sin\beta \sim g_R |\cos 2\beta|^{1/2}/2$.

Matrix M^2 yields an upper limit for the mass of the lighter Higgs boson (at least if fine-tuning conditions (16) are satisfied):

$$\begin{aligned} m_h^2 &\leq \frac{1}{2}(g_L^2 + g_R^2) M_L^2 \cos^2 2\beta - \frac{1}{2(g_R^2 + g_{B-L}^2)} M_L^2 \cos^2 2\beta \\ &\quad \times (g_R^2 + 4\lambda_\nu^2 \sin^2\beta x^2/\cos 2\beta)^2 + \mathcal{O}(M_{SUSY}^4/M_R^2) \\ &= m_{Z_L}^2 \cos^2 2\beta + \frac{1}{g_R^2 + g_{B-L}^2} 4\lambda_\nu^2 \sin^2\beta x^2 M_L^2 (-\cos 2\beta) \\ &\quad \times (g_R^2 - 2\lambda_\nu^2 \sin^2\beta(-\cos 2\beta) x^2) + \mathcal{O}(M_{SUSY}^4/M_R^2), \end{aligned} \tag{20}$$

where we have used $m_{Z_L}^2 = \frac{1}{2}(g_L^2 + g_Y^2) M_L^2$. At the limit of no R-parity breaking ($x=0$) and large right-handed scale ($M_L \ll M_R$) the mass limit reduces to the MSSM result.

B. Triplet Higgs bosons

The model 2b contains phenomenologically very interesting triplet Higgs fields. The masses of $SU(2)_L$ triplets Δ_L and δ_L are the free parameters of the theory: at the supersymmetric limit their mass is given by the mu term $\mu_{\Delta L}$. The masses of $SU(2)_R$ triplet fields Δ_R and δ_R are, on the other hand, strongly constrained by the spontaneous symmetry-breaking mechanism. One of the most exciting predictions specific to the left-right models is the existence of the doubly charged Higgs fields. The doubly charged Higgs field could be very light, and they can potentially be seen at LHC or at the planned electron-positron linear collider [15].

Combining Eq. (5) with results on Higgs boson mass limits presented in Appendix D, Eqs. (D3) and (D4), one finds

$$4f_R^2 v_{\Delta_R}^2 \leqslant f_R A_{f_R} v_{\Delta_R} + f_R \mu_{\Delta R} v_{\delta_R} \leqslant 8 f_R^2 \left(v_{\Delta_R}^2 - \frac{1}{3} v_{\delta_R}^2 \right), \tag{21}$$

where terms of order $\mathcal{O}(M_{SUSY}^2)$ have been ignored.

The minimization conditions of the scalar potential

$$\frac{1}{v_{\Delta_R}} \frac{\partial V}{\partial v_{\Delta_R}} = 2\mu_{\Delta_R}^2 + 2B_{\Delta_R}\mu_{\Delta_R}\frac{v_{\delta_R}}{v_{\Delta_R}} + 4(v_{\Delta_R}^2 - v_{\delta_R}^2)\left(4f_R^2 - \frac{f_R A_{f_R}}{v_{\Delta_R}}\right) + \mathcal{O}(M_{SUSY}^2) = 0, \tag{22}$$

$$\frac{1}{\sigma_R}\frac{\partial V}{\partial \sigma_R} = -4f_R A_{f_R} v_{\Delta R} - 4 f_R \mu_{\Delta R} v_{\delta_R} + 8 f_R^2 (2v_{\Delta_R}^2 - v_{\delta_R}^2) + \mathcal{O}(M_{SUSY}^2) = 0, \tag{23}$$

can be realized only if

$$|\mu_{\Delta R}| \sim |f_R v_{\Delta_R}| \equiv |f_R v_R| \quad \text{or} \quad |\mu_{\Delta R}|, |f_R v_R| \lesssim M_{SUSY}. \tag{24}$$

Combining Eqs. (21) and (23) it follows from the minimization of the potential that

$$|\mu_{\Delta R}| \lesssim M_{SUSY} \quad \text{and} \quad |f_R v_{\Delta_R}| = |f_R v_R| \sim M_{SUSY}. \tag{25}$$

These conditions are similar to Eqs. (20), which were obtained by requiring no fine tuning. Because the mu term $\mu_{\Delta R}$ is constrained to be of the order of the supersymmetry breaking scale, there are only two heavy ($m \gg M_{SUSY}$) Higgs fields: one neutral scalar field

$$\frac{1}{N} \times [2v_{\Delta_R}\mathrm{Re}(\Delta_R^0) - 2v_{\delta_R}\mathrm{Re}(\delta_R^0) - \sigma_R \mathrm{Re}(\tilde{\nu}_R)], \tag{26}$$

with mass $m^2 \simeq \frac{1}{2}(g_R^2 + g_{B-L}^2)(4v_{\Delta_R}^2 + 4v_{\delta_R}^2 + \sigma_R^2) \simeq m_{Z_R}^2$, and a charged Higgs field

$$\frac{1}{N} \times (2v_{\Delta_R} v_{\delta_R} \delta_R^{\pm} - (\sigma_R^2 + 2v_{\delta_R}^2)\Delta_R^{\pm} + \sqrt{2}\sigma_R v_{\delta_R} \tilde{e}_R^{\pm}), \tag{27}$$

with mass $m^2 \simeq \frac{1}{2} g_R^2 (2v_{\Delta_R}^2 + 2v_{\delta_R}^2 + \sigma_R^2) \simeq m_{W_R}^2$.

If we would have extended the Higgs sector by $U(1)_{B-L}$ singlet $SU(2)_R$ triplet or if we would have had some nonrenormalizable operators in the superpotential, we would have in supersymmetric minimum two Higgs fields at the right-handed scale v_R, while most of the scalar degrees of freedom would have a mass around $v_R^2/M_{Planck} \gg M_{SUSY}$, with v_R being at least 10^{10} GeV in nonrenormalizable model [13].

C. Additional Higgs doublets and *CP* violation

In models 1b and 2b the spectrum of Higgs bosons is quite large. They have total of four $SU(2)_L$ Higgs doublets. Two of them correspond to the MSSM doublets related to the electroweak symmetry breaking. The other two extra Higgs doublets can induce dangerous flavor-changing neutral currents, that would result in unacceptably large mass splitting and CP violation for K^0, D^0, and B^0 mesons. The limits on CP violation can set a lower limit of $\mathcal{O}(20\ \text{TeV})$ to the mass of the neutral flavor changing Higgs bosons ϕ_{12}^0 and ϕ_{21}^0 [12].

The CP-violating processes can be suppressed by a suitable definition of left-right symmetry. There are two possible ways to define the left-right symmetry in terms of the quark Yukawa matrices (see Appendix E):

$$\lambda_d = \pm \lambda_d^T, \quad \lambda_u = \pm \lambda_u^T \tag{28}$$

and

$$\lambda_d = e^{i\alpha}\lambda_d^\dagger, \quad \lambda_u = e^{i\beta}\lambda_u^\dagger. \tag{29}$$

The contribution of the mass matrices to the strong CP phase is at tree level arg Det$(M_u M_d)$, where M_u and M_d are mass matrices for the up and down quarks, respectively. This contribution to the strong CP phase would automatically vanish, if the Yukawa matrices are Hermitian, as in symmetry defined by Eq. (29), and if the vacuum expectation values of the Higgs bosons are real [16].

The extra Higgs bosons contribute to flavor-changing neutral currents (FCNC's) in K-$\bar{K}$ mixing. These contributions can set a lower limit of $\mathcal{O}$(20 TeV) to the mass of the extra Higgs bosons [12]. The contribution to the phase term is proportional to $\mathrm{Im}((V_L^* D_d V_R^\dagger)_{uc}(V_L^* D_d V_R^\dagger)_{cu}^*)$, where V_L and V_R are the two Cabibbo-Kobayashi-Maskawa present in the left-right-models, and D_d is diagonal matrix $D_d = \mathrm{diag}(m_d, m_s, m_b)$. If the model obeys symmetry of Eq. (28) the imaginary phase term vanishes. In this case the model is invariant under left-right transformation defined by

$$\phi_k \leftrightarrow -\tau_2 \phi_k^T \tau_2, \quad Q_L \leftrightarrow Q_R, \quad L_L \leftrightarrow L_R. \tag{30}$$

IV. COMPOSITION AND MASS OF LEPTONS

If the sneutrino has a nonvanishing vacuum expectation value then the Higgs bosons will mix with slepton fields and physical neutrinos or charged leptons will in general be mixtures of gauginos, Higgsinos, and lepton interaction eigenstates. In the following we will describe the lepton sector in models 2a and 2b. Similar results would apply also for models 1a and 1b.

The mass matrices are quite large. In Appendix F we present some approximative methods to compute the masses and compositions of the lightest charginos and neutralinos (the physical leptons). In this section we will just discuss the results.

A. Model 2a

The chargino and neutralino mass Lagrangian is of the form (see, e.g., [17])

$$\mathcal{L}=-\frac{1}{2}\begin{pmatrix}\Psi^{+T} & \Psi^{-T}\end{pmatrix}\begin{pmatrix}0 & X^T\\ X & 0\end{pmatrix}\begin{pmatrix}\Psi^{+}\\ \Psi^{-}\end{pmatrix} -\frac{1}{2}\Psi^{0T}Y\Psi^{0}+\text{H.c.} \tag{31}$$

In model 2a, $\Psi^{+T}=(-i\lambda_L^+,\tilde{\phi}_{22}^+,e_R^+)$, $\Psi^{-T}=(-i\lambda_L^-,\tilde{\phi}_{11}^-,e_L^-)$, and

$$X=\begin{pmatrix} M_L & g_L v_u & 0\\ g_L v_d & \mu_\phi & \lambda_e\sigma_L\\ g_L\sigma_L & -\lambda_\nu\sigma_R & -\lambda_e v_d\end{pmatrix}. \tag{32}$$

For neutralinos $\Psi^{0T}=(-i\lambda_L^0,-i\lambda_R^0,-i\lambda_{B-L}^0,\tilde{\phi}_{11}^0,\tilde{\phi}_{22}^0,\tilde{\Delta}_R^0,\tilde{\delta}_R^0,\nu_L,\nu_R)$. The upper triangle of symmetric matrix $Y=Y^T$ is

$$Y=\begin{pmatrix}
M_L & 0 & 0 & \frac{1}{\sqrt{2}}g_L v_d & -\frac{1}{\sqrt{2}}g_L v_u & 0 & 0 & \frac{1}{\sqrt{2}}g_L\sigma_L & 0\\
 & M_R & 0 & -\frac{1}{\sqrt{2}}g_R v_d & \frac{1}{\sqrt{2}}g_R v_u & \sqrt{2}g_R v_{\Delta_R} & -\sqrt{2}g_R v_{\delta_R} & 0 & -\frac{1}{\sqrt{2}}g_R\sigma_R\\
 & & M_{B-L} & 0 & 0 & -\sqrt{2}g_{B-L}v_{\Delta_R} & \sqrt{2}g_{B-L}v_{\delta_R} & 0 & \frac{1}{\sqrt{2}}g_{B-L}\sigma_R\\
 & & & 0 & -\mu_\phi & 0 & 0 & 0 & 0\\
 & & & & 0 & 0 & 0 & \lambda_\nu\sigma_R & \lambda_\nu\sigma_L\\
 & & & & & 0 & \mu_{\Delta R} & 0 & -2f_R\sigma_R\\
 & & & & & & 0 & 0 & 0\\
 & & & & & & & 0 & \lambda_\nu v_u\\
 & & & & & & & & -2f_R v_{\Delta_R}
\end{pmatrix}. \tag{33}$$

Let us define a dimensionless parameter $\tan\alpha_L$ that describes the strength of R-parity breaking couplings in model 2a:

$$\tan\alpha_L=\frac{\lambda_\nu\sigma_R}{\mu_\phi}. \tag{34}$$

The composition of physical charged lepton is then

$$\tau=\begin{pmatrix}\sin\alpha_L\tilde{\phi}_{11}^-+\cos\alpha_L e_L^-\\ \overline{e_R^+}\end{pmatrix}, \tag{35}$$

and mass is given by

$$m_\tau=|\lambda_e(\sin\alpha_L\sigma_L-\cos\alpha_L v_d)|. \tag{36}$$

The composition of physical neutrino is

$$\nu_\tau=\sin\alpha_L\tilde{\phi}_{11}^0+\cos\alpha_L\nu_L, \tag{37}$$

An approximation for neutrino mass can be calculated using methods described in Appendix F. Instead of giving the neutrino mass formula in its complete form we present here the result at the limit of large right-handed scale ($v_{\Delta_R}\gg M_{SUSY}$):

$$m_{\nu_\tau}\simeq\left|\frac{1}{2}\left[\frac{g_L^2}{M_L}+\left(\frac{M_{B-L}}{g_{B-L}^2}+\frac{M_R}{g_R^2}\right)^{-1}\right]\times(\cos\alpha_L\sigma_L+\sin\alpha_L v_d)^2-\frac{\lambda_\nu^2 v_u^2\cos^2\alpha_L}{2f_R v_{\Delta_R}}\right|. \tag{38}$$

Equation (38) is a reasonable approximation of the eigenvalue of the full mass matrix, since the v_{Δ_R} is expected to be at least at multi-TeV range. At the limit of no R-parity breaking ($\sigma_R=\sigma_L=0$) the neutrino mass formula reduces to the normal seesaw relation $m_{\nu_\tau}=\lambda_\nu^2 v_u^2/(2f_R v_{\Delta_R})$. Because of the constraint $f_R v_R\sim M_{SUSY}$ one would expect the neutrino mass always to be of the order m_τ^2/M_{SUSY}.

The sneutrino VEV's σ_L contribute also to the neutrino masses. At the limit of vanishing Yukawa couplings $\lambda_\nu=\lambda_e=0$ and universal gaugino masses $M=M_L=M_R$

$=M_{B-L}$ the neutrino mass can be approximated by Eq. (38). Using the current experimental limits on neutrino masses [18],

$$m_{\nu_e}<10\ \text{eV},\quad m_{\nu_\mu}<0.17\ \text{MeV},\quad m_{\nu_\tau}<18\ \text{MeV},\tag{39}$$

one obtains the following upper limits for the sneutrino VEV's $\sigma_{Lk}=\langle\tilde{\nu}_{Lk}\rangle$:

$$|\sigma_{Le}|<0.004\ \text{GeV}\left(\frac{M}{\text{TeV}}\right)^{1/2},$$

$$|\sigma_{L\mu}|<0.6\ \text{GeV}\left(\frac{M}{\text{TeV}}\right)^{1/2},$$

$$|\sigma_{L\tau}|<6\ \text{GeV}\left(\frac{M}{\text{TeV}}\right)^{1/2}.\tag{40}$$

applying to all models discussed in this work.

Taking the limits on neutrino masses (39) into account one can constrain the angle α_L for lepton family (at limit $\sigma_L=0$ and when the gaugino contribution dominates the neutrino masses):

$$|\sin\alpha_{Le}|\lesssim\frac{7\times10^{-5}}{\cos\beta}\left(\frac{M_{gaugino}}{\text{TeV}}\right)^{1/2},$$

$$|\sin\alpha_{L\mu}|\lesssim\frac{0.009}{\cos\beta}\left(\frac{M_{gaugino}}{\text{TeV}}\right)^{1/2},$$

$$|\sin\alpha_{L\tau}|\lesssim\frac{0.1}{\cos\beta}\left(\frac{M_{gaugino}}{\text{TeV}}\right)^{1/2}.\tag{41}$$

For the third lepton family the mixing is unrestricted for large values of $\tan\beta\gtrsim10$.[1]

B. Model 2b

The main difference between models 2a and 2b comes at the chargino sector: charged lepton can have components from the $SU(2)_R$ gaugino field and $SU(2)_R$ triplet Higgsino fields. The composition of the charged lepton is

$$\tau=\begin{pmatrix}\sin\alpha_L'\cos\alpha_L''\tilde{\phi}_{11}^-+\sin\alpha_L'\sin\alpha_L''\tilde{\phi}_{21}^-+\cos\alpha_L'e_L^-\\ \sin\alpha_R\cos\alpha_R'\overline{(-i\lambda_R^+)}+\sin\alpha_R\sin\alpha_R'\overline{\tilde{\delta}_R^+}-\cos\alpha_R\overline{e_R^+}\end{pmatrix}.\tag{42}$$

The angles α_R, α_L', and α_L'' are defined in Appendix G.

Due to approximate $SU(2)_L$ symmetry the physical neutrino is similar to the left-handed part of the physical charged lepton:

$$\nu_\tau=\sin\alpha_L'\cos\alpha_L''\tilde{\phi}_{11}^0+\sin\alpha_L'\sin\alpha_L''\tilde{\phi}_{21}^0+\cos\alpha_L'\nu_L.\tag{43}$$

The $SU(2)_R$ gaugino component in the right-handed part of the physical charged lepton is phenomenologically interesting: a large gaugino component in the physical lepton will result to lepton-number violating Yukawa operators that are specific for SUSYLR models. At the limit of the large right-handed scale $v_R\gg M_{SUSY}$ and setting $v_{\delta_R}=0$ and $M_R=\mu_{\Delta_R}=M_{SUSY}$, one has in leading order

$$\tan\alpha_R=\left(\frac{g_R\sigma_R}{M_{SUSY}}\right)^2,\quad \tan\alpha_R'=\frac{g_R\sigma_R}{M_{SUSY}}.\tag{44}$$

At this limit the right-handed part of the physical lepton is composed mostly of triplet Higgsinos ($\tilde{\delta}_R^+$) and $SU(2)_R$ gauginos ($-i\lambda_R^+$). The gaugino component in physical lepton can thus be quite large for moderately large sneutrino VEV σ_R.

V. FERMION COUPLINGS TO BOSONS

The physical processes where R-parity violation manifests itself will most probably include fermions. In this last section we discuss the Yukawa couplings and anomalous gauge couplings of the quarks and leptons.

A. Coupling to Higgs boson

The chargino mass Lagrangian can be written in the form

$$\mathcal{L}=-\Psi^{-T}X\Psi^{+}+\text{H.c.}=-\chi^{-T}D\chi^{+}+\text{H.c.},\tag{45}$$

where D is a diagonal positive definite matrix and $\chi^+=V\Psi^+$ and $\chi^-=U\psi^-$ and $X=X_0+X_1$ is the chargino mass matrix. X_1 contains all terms that are proportional to the VEV's that transform nontrivially under $SU(2)_L$, while X_0 contains all terms proportional to the supersymmetry breaking parameters and $SU(2)_L$ singlet VEV's.

We define unitary matrices U_0 and V_0 to be such that $D_0=U_0^*X_0V_0^\dagger$ is a diagonal matrix with non-negative entries, and $(D_0)_{11}=0$ (X_0 has in our case one zero eigenvector that corresponds to the physical lepton mass eigenstate).

At the limit of vanishing anomalous charged lepton coupling to Z_L boson the mass of the physical lepton is

$$m_1\approx(U_0^*X_1V_0^\dagger)_{11}.\tag{46}$$

At decoupling limit the lightest Higgs boson is [19]

$$h=\frac{1}{v}\sum_k(\langle\text{Re}\,\phi_k\rangle\text{Re}\,\phi_k+\langle\text{Im}\,\phi_k\rangle\text{Im}\,\phi_k),\tag{47}$$

where ϕ_k are all scalar doublet fields of the theory and $v^2=\Sigma_k\langle\phi_k^*\phi_k\rangle$. In our case this is equivalent to Eq. (17).

A tree-level Lagrangian describing the coupling of the lightest chargino (the charged physical lepton) $\chi_1^\pm(\sim\tau)$ to the Higgs boson h is [7]

[1] These constraints could be relaxed if the gaugino and triplet contributions to the neutrino mass were tuned to cancel out.

$$\mathcal{L} \simeq -\chi_1^-(U_0^* X_1 V_0^\dagger)_{11}\chi_1^+ h + \text{H.c}$$
$$= \left[1+\mathcal{O}\left(\frac{m_W}{M_H}\right)^2\right]\frac{m_\tau}{v}\tau^+\tau^- h + \text{H.c.} \quad (48)$$

At the decoupling limit the chargino coupling thus approaches the standard model prediction for the Higgs coupling, even if the physical lepton would be composed mainly of Higgsinos or gauginos.

B. Couplings to the weak currents

The lepton mass eigenstates are mixtures of lepton interaction eigenstates, higgsinos, and gauginos. All of these components do not necessarily have the same $SU(2)_L \times U(1)_Y$ quantum numbers as the standard model leptons. As a consequence, the lepton couplings to weak currents are nonuniversal and different from their standard model prediction.

The correction term for the neutral current couplings are given in Appendix F in Eq. (F12). The corrections to the axial and vectorial couplings are thus typically of order a^2/M_{SUSY}^2. Since the charged lepton mass is of the order $m_\tau \sim a$ and the neutrino mass is $m_\nu \sim a^2/M_{SUSY}$, typical perturbation to the axial or vectorial current would be $\delta A \sim \delta V \sim m_\nu^2/m_\tau^2 \sim m_\nu/m_{SUSY}$. If the neutrino masses are at their experimental upper bounds one would expect the perturbations to be

$$\delta A_e \sim 4\times 10^{-10}, \quad \delta A_\mu \sim 3\times 10^{-6}, \quad \delta A_\tau \sim 1\times 10^{-4}. \quad (49)$$

The experimental resolution is of the order 10^{-3}. In other words, the mass limits on neutrinos are generally more restricting than the limits obtained from the neutral current universality. Only the limit on τ family can be interesting, if the neutrino mass is close to its experimental bound and the model parameters are chosen appropriately.

The standard model prediction for the axial current is $A_\tau^{SM} = -\frac{1}{2}$. Assuming that two σ deviation from the standard model prediction is acceptable [18], the axial current can differ from the standard model prediction by

$$|\delta A_\tau| = |A_\tau - A_\tau^{MS}| < 0.0026. \quad (50)$$

One can derive an analytic expression for the deviation of the axial and vector current:

$$\left.\begin{matrix}\delta A\\ \delta V\end{matrix}\right\} = \delta L \mp \delta R = \frac{1}{2}(\sigma_L \cos\alpha_L + v_d \sin\alpha_L)^2 \times\left(-\frac{g_L^2}{M_L^2} \mp \frac{\lambda_\nu^2}{\mu_\phi^2}\right). \quad (51)$$

When compared to the expression for the neutrino mass (38), one sees that the deviation from the standard model prediction is typically less than $m_{\nu_\tau}/M_{gaugino}$. The anomalous coupling to weak current is thus practically always less than the experimental error in the measurement. (The anomalous coupling can, however, be large if the ratio λ_ν^2/μ_ϕ^2 is big enough: $|\mu_\phi| \ll |\lambda_\nu M_L/g_L|$.)

Similar result applies to charged weak current, since both physical neutrino and charged lepton mass eigenstates obey $SU(2)_L$ symmetry to a good accuracy. The $SU(2)_L$ breaking mixing angles in lepton mass eigenstates are typically suppressed by factor $\sqrt{m_\nu/M_{SUSY}}$, as shown above for neutral weak current.

C. *R*-parity breaking couplings

Most of the *R*-parity breaking couplings are suppressed either by the large right-handed scale, by nonobservation of heavy neutrinos or by experimental constraints on the universality of neutral and charged weak currents. There are, however, a limited set of dimension three operators that break *R* parity and that can be large. All *R* parity breaking Yukawa operators that couple to two standard model fermions and to a scalar field are listed.

For simplicity, only one lepton family is taken to have nonvanishing sneutrino VEV'(s). We denote with k the index of this family ($\langle\tilde{\nu}_{Rk}\rangle \neq 0$), with i an arbitrary lepton or quark family and with j an arbitrary lepton or quark family that satisfies $j \neq k$.

The physical leptons have a Higgsino component. The mixing in model 1a or 2a is proportional to angle α_L. This results in the following effective operators:

$$\mathcal{L}_{2a} = -\lambda_{di}\sin\alpha_L(\overline{d_i^c}P_L\nu_k\tilde{d}_{Ri} + \overline{d}_iP_L\nu_k\tilde{d}_{Li} - \overline{u_i^c}P_Le_k\tilde{d}_{Ri} - \overline{d}_iP_Le_k\tilde{u}_{Li}) + \text{H.c.} \quad (52)$$

The lepton-number-violating couplings are proportional to the down-quark Yukawa couplings. All couplings are parametrized by mixing angle $\tan\alpha_{Lk}$, which is constrained by neutrino masses [see Eq. (41)].

In model 2a one has also Higgsino $\tilde{\phi}_{21}^-$ components and $-i\lambda_R^+$ gaugino components mixed in the physical lepton mass eigenstate. These fields can induce couplings that are proportional to up-quark Yukawas and gauge coupling g_R:

$$\mathcal{L}_{2b} = -\sin\alpha_L'(\lambda_{di}\cos\alpha_L'' + \lambda_{ui}\sin\alpha_L'')(\overline{d_i^c}P_L\nu_k\tilde{d}_{Ri} + \overline{d}_iP_L\nu_k\tilde{d}_{Li} - \overline{u_i^c}P_Le_k\tilde{d}_{Ri} - \overline{d}_iP_Le_k\tilde{u}_{Li}) - g_R\sin\alpha_R\cos\alpha_R'\overline{u_i^c}P_Re_k\tilde{d}_{Ri} + \text{H.c.} \quad (53)$$

The last term in Eq. (53) is a unique lepton number violating coupling. It couples universally, with the same strength, to all (s)quark families. Further, it is not suppressed by the Yukawa couplings. It is thus the only large *R*-parity violating coupling that involves light quark and lepton families. Since the coupling is due to mixing of $SU(2)_R$ it is also a unique prediction of *R*-parity violating SUSYLR models

The *R*-parity violating operators involving (s)leptons are similar to those involving quarks. The only difference is that some operators are cancelled out, if all sleptons involved are from the family having nonzero sneutrino VEV. The operator proportional to the gauge coupling involves heavy right-handed neutrino, so it will not be listed here. The operators in model 2b are the following:

$$\mathcal{L}=-\sin\alpha_L'(\lambda_{ej}\cos\alpha_L''+\lambda_{\nu j}\sin\alpha_L'')\times(\overline{e_j^c}P_L\nu_k\tilde{e}_{Rj}+\overline{e_j}P_L\nu_k\tilde{e}_{Lj}-\overline{\nu_j^c}P_Le_k\tilde{e}_{Rj}-\overline{e}_jP_Le_k\tilde{\nu}_{Lj})-\sin\alpha_L'\cos\alpha_L'(\lambda_{ek}\cos\alpha_L''+\lambda_{\nu k}\sin\alpha_L'')\times(\overline{e_k}P_L\nu_k\tilde{e}_{Lk}-\overline{e_k}P_Le_k\tilde{\nu}_{lk}). \quad (54)$$

The result for model 2a is obtained from Eq. (54) by replacing $\sin\alpha_L'(\lambda_{ei}\cos\alpha_L''+\lambda_{\nu j}\sin\alpha_L'')$ by $\sin\alpha_L\lambda_{ei}$.

The trilinear *R*-parity breaking couplings in models 1a and 2a are similar to those in MSSM with sneutrino VEV's. Models 1b and 2b have two distinct features: There is proportionality to down *and* up quark Yukawa matrix λ_u due to Higgsino components $\tilde{\phi}_{12}^{0,\pm}$ in the physical leptons; there is a contribution due to the $SU(2)_R$ gaugino in the right-handed part of the physical lepton

$$\mathcal{L}=-g_R\sin\alpha_R\sin\alpha_R'\tilde{d}_{Ri}\overline{u_i^c}P_Re_k+\text{H.c.} \quad (55)$$

The contribution due to gaugino is universal for all (s)quark families. The mixing angles α_R and α_R' are *a priori* free parameters, while the left-handed mixing angles (α_L, α_L' and α_L'') are constrained by the neutrino masses (41).

The *R*-parity breaking vertex proportional to the gauge coupling g_R involves only $SU(2)_L$ singlet fields. The operator could be directly measured at process $e_k^+\overline{u}\to\tilde{d}_R^*$ or $e^-u\to\tilde{d}_R$. The latter process could be detected in HERA, if the electron sneutrino has a nonvanishing VEV $\sigma_{Re}\neq 0$ and the down-squark $\tilde{d}_R$ is near the experimental lower limit on its mass ($\sim$200 GeV).

A more stringent limit, if the electron sneutrino has a nonvanishing VEV $\sigma_{Re}\neq 0$, is given by nonobservation of neutrinoless double β decay. The limit obtained from the lower bound on the lifetime of ^{76}Ge gives [20]

$$|g_R\sin\alpha_{Re}\cos\alpha_R'|\lesssim 0.07\left(\frac{\tilde{m}_{d_R}}{\text{TeV}}\right)^2\left(\frac{M_{gluino}}{\text{TeV}}\right)^{1/2}, \quad (56)$$

where only the graph involving gluino and down-squarks $\tilde{d}_R$ has been taken into account.

VI. CONCLUSION

I have analyzed a set of minimal models that obey the left-right gauge symmetries and in which the *R*-parity is broken spontaneously by a VEV of a sneutrino. In two of our models (1a and 1b), in which the right-handed scale is close to the supersymmetry breaking scale, the $SU(2)_R$ triplet superfields are not needed to have an acceptable spontaneous symmetry breaking pattern. The VEV of right-handed neutrino alone is sufficient to make the right-handed gauge bosons heavy enough. I have analyzed Higgs sectors of these models. The Higgs sector is characterized by one scalar that at the decoupling limit is like the standard model Higgs boson. The upper limit for its mass can be pushed by radiative corrections as high as 150−200 GeV. In model 2b at the limit of the large right-handed scale there are always either light doubly charged scalar degrees of freedom or a light neutral singlet degree of freedom.

I have found analytic expressions for masses and mixings of the neutral and charged leptons. In Appendix F, I present a general method to calculate the mass eigenvalues and eigenvectors for large fermion mass matrices. The experimental bounds on neutrino masses set strict limits on the left-handed sneutrino VEV and on the anomalous couplings to the neutral weak current. The deviations to the couplings with the neutral weak currents are expected to be too small to be observed.

The *R*-parity breaking trilinear couplings that are unsuppressed by the low neutrino masses are listed. In model 1a and 2a the lepton number violating trilinear couplings are always proportional to the mixing angle $\sin\alpha_L$ and the Yukawa coupling of corresponding quark or lepton family.

In $SU(2)_R$ models the mixing of right-handed part of the charged lepton with the $SU(2)_R$ gaugino introduces for a universal *R*-parity breaking coupling that is proportional to the gauge coupling g_R. This coupling and *R*-parity breaking coupling proportional to the up-quark mass matrix can provide unique signature of $SU(2)_R\times U(1)_{B-L}$ gauge symmetry group.

APPENDIX A: SHAPE OF THE POTENTIAL AT RIGHT-HANDED SCALE

The scalar potential of models 1b or 2b expressed in terms of the right-handed field VEV's can be written as

$$V\simeq\frac{1}{8}(g_R^2+g_{B-L}^2)(\sigma_R^2+2v_{\delta_R}^2-2v_{\Delta_R}^2)^2+4f_R^2\sigma_R^2v_{\Delta_R}^2+(\mu_{\Delta R}v_{\delta_R}-f_R\sigma_R^2)^2+\mu_{\Delta R}^2v_{\Delta R}^2+AM_{SUSY}^2\sigma_R^2+BM_{SUSY}^2v_{\delta_R}^2+CM_{SUSY}^2v_{\Delta_R}^2+DM_{SUSY}\mu_{\Delta R}v_{\delta_R}v_{\Delta_R}+Ef_RM_{SUSY}v_{\delta_R}\sigma_R^2, \quad (A1)$$

where M_{SUSY} is the supersymmetry breaking scale and *A*, *B*, *C*, *D*, and *E* are some dimensionless parameters of order unity that depend on the soft supersymmetry breaking couplings. If we consider a simplified equation, where we take $\mu_{\Delta R}=D=E=0$, we can minimize the potential *V* analytically. There are three possible solutions for the global minimum of the potential *V* at the limit of small f_R, corresponding to large right-handed scale v_R. The first solution is the trivial solution

$$\sigma_R=v_{\delta_R}=v_{\Delta_R}=V_{MIN}=0. \quad (A2)$$

The second solution is (if $-B-C\geqslant 0$, $-2A+3B+2C\geqslant 0$ and $-2A+2B+C\geqslant 0$)

$$\sigma_R^2=\frac{-B-C}{4f_R^2}M_{SUSY}^2,$$

$$v_{\delta_R}^2=\frac{-2A+3B+2C}{8f_R^2}M_{SUSY}^2,$$

$$v_{\Delta_R}^2=\frac{-2A+2B+C}{8f_R^2}M_{SUSY}^2,$$

$$V_{MIN}=-\frac{(4A-3B-C)(B+C)}{16f_R^2}M_{SUSY}^4, \tag{A3}$$

and the third solution is (if $-2A-C\geqslant 0$)

$$\sigma_R^2=2v_{\Delta_R}^2=\frac{-2A-C}{12f_R^2}M_{SUSY}^2, \quad v_{\delta_R}^2=0,$$

$$V_{MIN}=-\frac{(2A+C)^2}{48f_R^2}M_{SUSY}^4. \tag{A4}$$

If, for example, the supersymmetry breaking parameters are chosen to be $A=-4$, $B=0$ and $C=1$, then solution (A4) is the global minimum. The VEV's are then

$$\sigma_R^2\simeq 2v_{\Delta_R}^2\simeq\frac{7}{12}\frac{M_{SUSY}^2}{f_R^2}\simeq v_R, \quad v_{\delta_R}^2\simeq 0. \tag{A5}$$

[In the case of model 2b and in this particular case the gauge couplings should obey $1<g_{B-L}^2/g_R^2<\frac{13}{7}$ for the global minimum not to break the residual $U(1)_{em}$ gauge symmetry.]

One would thus expect that the right-handed VEV's have the following pattern:

$$\sigma_R^2, v_{\Delta_R}^2, v_{\delta_R}^2\sim v_R^2 \quad \text{or} \quad \sigma_R^2, v_{\Delta_R}^2\sim v_R^2, v_{\delta_R}^2\sim\mathcal{O}(M_{SUSY}^2),$$

$$|D|=|\sigma_R^2+2v_{\delta_R}^2-2v_{\Delta_R}^2|\sim\mathcal{O}(M_{SUSY}^2). \tag{A6}$$

As a result of soft supersymmetry breaking couplings of the order $M_{SUSY}\simeq\mathcal{O}(1\text{ TeV})$, it is natural to have the sneutrino VEV σ_R of the order of right-handed scale $v_R\gg M_{SUSY}$: $\sigma_R\simeq\mathcal{O}(v_R)$. With full mass matrix, taking all parameters into account, this is indeed the case.

In the limit of large right-handed scale $v_R\gg M_{SUSY}$ the value of the potential at minimum is typically quite large: $V_{MIN}\sim M_{SUSY}^2 v_R^2\gg M_{SUSY}^4$. One could potentially have large quadratic corrections to the scalar mass terms. It is shown in Appendix B that the quadratic correctins are suppressed if the couplings obey certain relations.

APPENDIX B: FINE-TUNING CONSIDERATIONS

The quadratic radiative corrections δM^2 to the scalar masses are typically of the order

$$\delta M^2\sim\frac{\lambda^2}{8\pi^2}\delta\mu^2, \tag{B1}$$

where λ is some Yukawa or gauge coupling constant and $\delta\mu^2$ is typical mass difference between corresponding scalar and fermion degrees of freedom. If $\delta\mu^2$ is large ($\gg M_{SUSY}^2$) the radiative corrections to the scalar mass terms δM^2 could potentially also be large. In other words, one would have reintroduced the naturalness problem.

In supersymmetric minimum of model 2a all VEV's vanish. However, we require some of the VEV's to be much larger than the supersymmetry breaking scale. This could potentially result in large mass splitting between fermionic and bosonic degrees of freedom.

The part of the scalar potential related to the F terms is

$$V_F=\sum_k |F_k|^2, \tag{B2}$$

where $F_k=\partial W/\partial\phi_k$ denotes partial derivative of superpotential W with respect to a chiral superfield ϕ_k. The contribution of the F terms to the real scalar mass matrix $\tilde{M}_{ij}^2$ is

$$\begin{aligned}\sum_k \frac{1}{2}\left(\frac{\partial^2}{\partial\phi_i\partial\phi_j}-\frac{\delta_{ij}}{v_i}\frac{\partial}{\partial\phi_i}\right)\left(\frac{\partial W}{\partial\phi_k}\right)^2 &= \sum_k \left\langle\frac{\partial^2 W}{\partial\phi_i\partial\phi_k}\right\rangle\left\langle\frac{\partial^2 W}{\partial\phi_k\partial\phi_j}\right\rangle \\ &\quad +\sum_k \langle F_k\rangle\left\langle\frac{\partial^2 F_k}{\partial\phi_i\partial\phi_j}-\frac{\delta_{ij}}{v_i}\frac{\partial F_k}{\partial\phi_i}\right\rangle,\end{aligned} \tag{B3}$$

where v_i denotes the VEV of chiral superfield ϕ_i. The first sum in Eq. (B3) gives the supersymmetric mass terms that are similar to those in the neutralino mass matrix. The part proportional to F term F_k contributes to the mass splitting between scalar and fermion degrees of freedom.

From the scalar mass matrices one can see that the mass difference $\delta\mu^2$ in the present model (with a large σ_R) due to large F terms is restricted to $\delta\mu^2\lesssim M_{SUSY}^2$, providing that the model parameters obey the following relations:[2]

$$|\mu_\phi|\lesssim M_{SUSY}, \quad |\mu_{\Delta R}|\lesssim M_{SUSY},$$

$$|\lambda_\nu\sigma_R|\lesssim M_{SUSY}, \text{ and } |f_R v_R|\lesssim M_{SUSY}. \tag{B4}$$

Radiative corrections to the scalar potential should thus have no large quadratic corrections, even if we are in fact quite far from the supersymmetric minimum of the scalar potential.

[2]One could derive the limits (B4) also from minimization conditions of the scalar potential $\partial V/\partial\phi_k=0$ by requiring that the soft supersymmetry breaking terms are at most of the order of M_{SUSY}. Similar inequalities have been found in the case of Model 2b in [7].

TABLE I. Physical scalar mass eigenstates for a particular choice of parameters in model 2b ($v_{\Delta_R}=10^7$ GeV, $v_{\delta_R}=1.2\times10^6$ GeV, $\sigma_R=1.4\times10^7$ GeV, $\tan\beta=3$). The $SU(2)_L$ triplet fields Δ_L and δ_L have not been shown. They do not mix with the other scalar fields. Also the squarks and the first and second family sleptons have been left out. The model contains light [$\mathcal{O}(10\ \text{GeV})$] scalar and pseudoscalar degrees of freedom that are singlets under the standard model gauge group. There are always necessarily two heavy scalar degrees of freedom that have a mass of the order of the right-handed scale. The doubly charged scalar fields have in this particular case a mass around the supersymmetry breaking scale M_{SUSY}. The mass eigenstates are calculated at *tree level*, the radiative corrections would, e.g., increase the mass of the light Higgs doublet mass eigenstate.

Mass (TeV)	Composition
1.4×10^4	$-0.098\,\text{Re}(\delta_R^0)+0.815\,\text{Re}(\Delta_R^0)-0.572\,\text{Re}(\tilde\nu_R)$
9.3×10^3	$0.085\delta_R^\pm-0.707\Delta_R^\pm+0.702e_R^\pm$
9.6	$-0.695\phi_{21}^\pm-0.237\phi_{22}^\pm-0.639\phi_{11}^\pm-0.232\phi_{12}^\pm$
9.6	$-0.694\,\text{Re}(\phi_{11}^0)-0.237\,\text{Re}(\phi_{12}^0)+0.639\,\text{Re}(\phi_{21}^0)+0.232\,\text{Re}(\phi_{22}^0)$
9.6	$0.694\,\text{Im}(\phi_{11}^0)+0.237\,\text{Im}(\phi_{12}^0)+0.639\,\text{Im}(\phi_{21}^0)+0.231\,\text{Im}(\phi_{22}^0)$
9.2	$0.993\,\text{Im}(\delta_R^0)+0.119\,\text{Im}(\Delta_R^0)$
9.0	$0.993\,\text{Re}(\delta_R^0)+0.04\,\text{Re}(\Delta_R^0)-0.113\,\text{Re}(\tilde\nu_R)$
8.8	$0.993\delta_R^\pm-0.12e_R^\pm$
8.7	$0.988\delta_R^{\pm\pm}-0.154\Delta_R^{\pm\pm}$
6.2	$0.646\,\text{Re}(\phi_{11}^0)-0.291\,\text{Re}(\phi_{12}^0)+0.672\,\text{Re}(\phi_{21}^0)-0.215\,\text{Re}(\phi_{22}^0)$
6.2	$0.646\phi_{21}^\pm-0.291\phi_{22}^\pm-0.672\phi_{11}^\pm+0.215\phi_{12}^\pm$
6.2	$-0.646\,\text{Im}(\phi_{11}^0)+0.291\,\text{Im}(\phi_{12}^0)+0.672\,\text{Im}(\phi_{21}^0)-0.215\,\text{Im}(\phi_{22}^0)$
3.1	$-0.154\delta_R^{\pm\pm}-0.988\Delta_R^{\pm\pm}$
1.9	$0.025\phi_{21}^\pm+0.927\phi_{22}^\pm-0.375\phi_{11}^\pm+0.008\phi_{12}^\pm$
1.9	$0.025\,\text{Re}(\phi_{11}^0)+0.927\,\text{Re}(\phi_{12}^0)+0.375\,\text{Re}\,\phi_{21}^0-0.008\,\text{Re}(\phi_{22}^0)$
1.9	$0.025\,\text{Im}(\phi_{11}^0)+0.927\,\text{Im}(\phi_{12}^0)-0.375\,\text{Im}(\phi_{21}^0)+0.008\,\text{Im}(\phi_{22}^0)$
1.7	$\tilde\tau_L^\pm$
1.7	$\text{Re}(\tilde\nu_L)$
1.7	$\text{Im}(\tilde\nu_L)$
0.073	$-0.316\,\text{Re}(\phi_{11}^0)-0.949\,\text{Re}(\phi_{22}^0)$
0.039	$0.07\,\text{Re}(\delta_R^0)+0.579\,\text{Re}(\Delta_R^0)+0.813\,\text{Re}(\tilde\nu_R)$
0.009	$-0.068\,\text{Im}(\delta_R^0)+0.568\,\text{Im}(\Delta_R^0)+0.82\,\text{Im}(\tilde\nu_R)$

Another way to analyze fine tuning is to write the electroweak gauge boson masses in terms of model parameters at higher scale. In this case the minimization conditions for the potential yield at tree level:

$$\frac{g_R^2}{g_L^2}m_{W_L}^2\cos 2\beta=m_{\delta_R}^2+\mu_{\Delta R}^2+\frac{1}{2}(g_R^2+g_{B-L}^2)D+\frac{\mu_{\Delta R}}{v_{\delta_R}}(v_{\Delta_R}B_{\Delta R}-f_R\mu_{\Delta_R}\sigma_R^2),$$

$$\frac{g_R^2}{g_L^2}m_{W_L}^2\cos 2\beta=-m_{\Delta_R}^2-\mu_{\Delta R}^2+\frac{1}{2}(g_R^2+g_{B-L}^2)D-4f_R^2\sigma_R^2+\frac{-v_{\delta_R}\mu_{\Delta R}B_{\Delta R}+f_RA_{f_R}\sigma_R^2}{v_{\Delta_R}}. \tag{B5}$$

If the conditions in Eq. (B4) apply all terms in Eq. (B5) are of order $\mathcal{O}(M_{SUSY}^2)$ and there is no need for fine-tuned cancellations.

APPENDIX C: EXAMPLE MODEL

We provide a model in Table I.

APPENDIX D: HIGGS BOSON MASS LIMITS

The mass of the lightest neutral flavor changing Higgs boson, composed of ϕ_{12}^0 and ϕ_{21}^0, is bound by

$$\mathcal{O}(1-10\ \text{TeV})^2<M^2_{\phi_{12}^0,\phi_{21}^0}\leqslant-\cos^2 2\beta m_{W_L}^2-\frac{1}{2}g_R^2\cos 2\beta\times\left(D+\frac{2m_{W_L}^2\cos 2\beta}{g_L^2}\right)+\sigma_R^2((\lambda_e)^2\cos^2\beta-(\lambda_\nu)^2\sin^2\beta), \tag{D1}$$

where D is given in Eqs. (5) or (12) for the models 1b and 2b, respectively. One sees that one must either have a positive D term, $\frac{1}{2}g_R^2D>\mathcal{O}(1-10\ \text{TeV})^2$, or alternatively

$\lambda_e \cos\beta\sigma_R$ should have a value at least of the order of TeV (if λ_e is tau Yukawa coupling σ_R should be larger than about 100 TeV). Added together, in model 1b the sneutrino VEV has a lower limit $\sigma_R \gtrsim 3$ TeV (or equivalently $m_{W_R} \gtrsim 1$ TeV).

It turns out that *all* bidoublet Higgs bosons can have a mass of at most of the order M_{SUSY}: the only parameters that could make them heavier would be the parameters μ_ϕ^{ij}. However, the minimization conditions for bidoublet fields read (ignoring all terms of order M_{SUSY})

$$\frac{1}{v_u}\frac{\partial V}{\partial v_u} = 16(\mu_\phi^{11})^2 + 16(\mu_\phi^{12})^2 \sim M_{SUSY}^2,$$

$$\frac{1}{v_d}\frac{\partial V}{\partial v_u} = 16(\mu_\phi^{22})^2 + 16(\mu_\phi^{12})^2 \sim M_{SUSY}^2. \qquad \text{(D2)}$$

It follows that the μ terms, and consequently bidoublet Higgs masses, are also at most of the SUSY-breaking scale.[3]

One can derive the following upper bound to the mass-squared term of a neutral Higgs scalar from 3×3 submatrix of the full mass matrix of models 2a and 2b that involves the fields Δ_R^0, δ_R^0 and $\tilde{\nu}_R$:

$$\begin{aligned} 0 &< (v_{\Delta_R}^2 + v_{\delta_R}^2 + \sigma_R^2) M^2_{\Delta_R^0,\delta_R^0\tilde{\nu}_R} \\ &\leqslant \frac{1}{2}(g_R^2 + g_{B-L}^2)D^2 + 4f_R^2\sigma_R^2(\sigma_R^2 + 4v_{\Delta_R}^2) \\ &\quad - 3f_R A_{f_R}\sigma_R^2 v_{\Delta_R} - 3f_R\mu_{\Delta R} v_{\delta_R}\sigma_R^2. \end{aligned} \qquad \text{(D3)}$$

The doubly charged Higgs boson of the model 2b has the following upper bound on its mass:

$$\begin{aligned} 0 &< (v_{\Delta_R}^2 + v_{\delta_R}^2) M^2_{\Delta_R^{\pm\pm},\delta_R^{\pm\pm}} \\ &\leqslant g_R^2(v_{\Delta_R}^2 - v_{\delta_R}^2)D + f_R A_{f_R}\sigma_R^2 v_{\Delta_R} - 4f_R^2\sigma_R^2 v_{\Delta_R}^2 \\ &\quad + f_R\mu_{\Delta R} v_{\delta_R}\sigma_R^2. \end{aligned} \qquad \text{(D4)}$$

APPENDIX E: MOST GENERAL DISCRETE LEFT-RIGHT SYMMETRY

The superfield content of models 2a and 2b are explicitly left-right-symmetric, if also the left-handed triplets Δ_L and δ_L are taken into account. The quark fields and bidoublet Higgs fields transform in left-right transformations as follows:

$$Q_L \to U_L Q_L, \quad \tilde{Q}_L \to U_L \tilde{Q}_L, Q_R \to U_R Q_R,$$

$$\tilde{Q}_R \to U_R \tilde{Q}_R, \quad \phi_i \to U_L \phi_i U_R^\dagger, \quad \tilde{\phi}_i \to U_L \tilde{\phi}_i U_R^\dagger, \qquad \text{(E1)}$$

where the charge-conjugated fields have been used

$$\tilde{Q}_{L,R} = i\tau_2 Q_{L,R}^* \text{ and } \tilde{\phi}_i = -i\tau_2\phi_i^* i\tau_2, \qquad \text{(E2)}$$

and the left-right transformation is defined as

$$U_{L,R} = \exp\left(-\frac{1}{2} i \epsilon_{L,R}^k \tau_k\right). \qquad \text{(E3)}$$

The discrete $\mathcal{Z}_2$ left-right transformation means that the model, including the quark mass term, remains invariant under the interchange of U_L and U_R, and that two consequent left-right transformations reduce to the identity:

$$\mathcal{L} = -\lambda_i^{ab} Q_{La}^T i\tau_2\phi_i Q_{Rb} - \lambda_i^{ab*}\tilde{Q}_{La}^T i\tau_2\tilde{\phi}_i\tilde{Q}_{Rb}. \qquad \text{(E4)}$$

Clearly, as the gauge operators $U_{L,R}$ are swapped $U_L \leftrightarrow U_R$ the fields must transform as follows:

$$Q_L^a \to U_{1b}^a Q_R^b + V_{1b}^a \tilde{Q}_R^b,$$

$$Q_R^a \to U_{2b}^a Q_L^b + V_{2b}^a \tilde{Q}_L^b,$$

$$\phi_i \to X_i^j i\tau_2\phi_j^T i\tau_2 + Y_i^j i\tau_2\tilde{\phi}_j^T i\tau_2. \qquad \text{(E5)}$$

Since there are no charge-conjugate fields in the superpotential, one must have either $U_i = X = 0$ or $V_i = Y = 0$. By a suitable redefinition of the quark field Q_L, one can set $U=1$ or $V=1$. Matrix X or Y can, in principle, be any unitary 2×2 matrix that satisfies $X^2=1$ or $YY^*=1$. Only cases where matrices X and Y are diagonal will be considered.

There are thus two ways to define the left-right-symmetry in terms of quark Yukawa matrices:

$$(a) \quad V_i = Y = 0: \quad \lambda_d = X_d^d \lambda_d^T, \quad \lambda_u = X_u^u \lambda_u^T;$$

$$(b) \quad U_i = X = 0: \quad \lambda_d = Y_d^d \lambda_d^\dagger, \quad \lambda_u = Y_u^u \lambda_u^\dagger; \qquad \text{(E6)}$$

where $X_u^u, X_d^d = \pm 1$ and $|Y_u^u|, |Y_d^d| = 1$ is an arbitrary phase. If the Lagrangian of the model, including the gauge couplings and triplet Higgs fields, obey these left-right symmetries, the symmetry is also preserved in the renormalization-group running of the model.

APPENDIX F: MASS EIGENVALUES AND EIGENVECTORS FOR FERMIONS

The chargino and neutralino mass matrices are typically quite large and cannot be solved analytically. The fermion mass matrix is generally of the form

[3]The flavor changing Higgs doublets could be made to have a mass around the right-handed scale v_R by introducing nonrenormalizable operator $1/M_* \mathrm{Tr}\,\phi_1 i\tau_2\phi_2^T i\tau_2 \mathrm{Tr}\,\Delta_R\delta_R$ to the superpotential [4].

$$\mathcal{L} = -\frac{1}{2}\Psi^T Y \Psi + \text{H.c.} = -\frac{1}{2}\chi^T D \chi + \text{H.c.}, \tag{F1}$$

where Ψ is a vector of Weyl spinors and $Y = Y^T$ is a symmetric mass matrix. D is a diagonal mass matrix with non-negative entries and $\chi = N\Psi$ are the fermion mass eigenstates. The unitary matrix N satisfies $N^* Y N^\dagger = D$, or $D^2 = N Y^\dagger Y N^\dagger$.

For Dirac fermions the mass matrix Y is of the form

$$Y = \begin{pmatrix} 0 & X^T \\ X & 0 \end{pmatrix}. \tag{F2}$$

The diagonalizing matrix N is

$$N = \begin{pmatrix} V & -U \\ V & U \end{pmatrix}, \tag{F3}$$

where V and U are unitary matrices such that $D_D = U^* X V^\dagger$ is a diagonal matrix with non-negative entries (see, e.g., [17] for further discussion). The eigenvectors of the Dirac mass matrix come always in pairs having opposite mass eigenvalues. Although the derivation in this section is given for Majorana spinors, the generalization to Dirac spinors (i.e., charginos) is straightforward.

In our case the mass matrix Y can always be decomposed into two parts $Y = Y_0 + Y_1$, where Y_0 contains all supersymmetry breaking terms and all terms that are proportional to vacuum expectation values that are singlet under $SU(2)_L \times U(1)_Y$. $Y_0^\dagger Y_0$ is thus always constructed of block-diagonal submatrices of constant hypercharge Y^{HC}. Y_1 contains all terms that are proportional to VEV's that break the standard model gauge group (in our models v_u, v_d, and σ_L).

In all our cases the matrix Y_0 has at least one zero eigenvalue that approximately corresponds to the physical lepton. The mass of the lepton is induced by the (small) terms in matrix Y_1. Our idea is first to transform to basis where zero eigenvectors of matrix Y_0 are unit vectors. It is enough for purposes of this work to assume that Y_0 has only one zero eigenvalue. In the end, I give a general result for arbitrary number of zero eigenvectors of Y_0. First we transform to the basis where the physical lepton eigenvectors are approximately unit vectors $\tilde{v}_0^T = (1,0,\ldots,0)$. To this end an unitary matrix $\hat{N}_0$ is defined that satisfies[4]

$$\tilde{Y}_0 = (\hat{N}_0^* Y_0 \hat{N}_0^\dagger)_{ij} = 0, \quad i = 1 \,\text{or}\, j = 1. \tag{F4}$$

We further define matrices $\tilde{Y}_1 = \hat{N}_0^* Y_1 \hat{N}_0^\dagger$, $\tilde{Y} = \hat{N}_0^* Y \hat{N}_0^\dagger$, $a_i = (\hat{N}_0^* Y_1 \hat{N}_0^\dagger)_{1i}$ and $\hat{Y}_0 = (\tilde{Y}_0)_{\hat{1}\hat{1}}$, where $A_{\hat{i}\hat{j}}$ denotes matrix A with row i and column j removed.

[4]To find matrix $\hat{N}_0$ we need to find the zero eigenvector of $Y_0^\dagger Y_0$. One can always find an analytical expression for inverse of an arbitrary matrix. n zero eigenvectors of matrix Y_0 can be found from the basis spanned by $\lim_{\varepsilon \to 0} \varepsilon^n (Y_0 + \varepsilon \mathbf{1})^{-1}$.

We develop the mass of the lightest eigenvector into series with respect to the eigenvalues of Y_1. There are many ways to do it; the simplest expression is obtained by using determinants. The lepton mass m is

$$\begin{aligned} m &= \frac{|D|}{|D_{\hat{1}\hat{1}}|} \\ &= \frac{|N^*(Y_0+Y_1)N^\dagger|}{|(N^*(Y_0+Y_1)N^\dagger)_{\hat{1}\hat{1}}|} \\ &= \begin{cases} a_1 + \mathcal{O}(Y_1^2), & a_1 \neq 0, \\ \displaystyle\sum_{i,j\neq 1} (-1)^{i+j} a_i a_j \frac{|(\hat{Y}_0)_{\widehat{i-1},\widehat{j-1}}|}{|\hat{Y}_0|} + \mathcal{O}(Y_1^3), & a_1 = 0. \end{cases} \end{aligned} \tag{F5}$$

The ratio of derivatives in Eq. (F5) is simplified by the fact that $\hat{Y}_0$ is a block-diagonal matrix. If the blocks are small enough the ratios reduce to quite simple expressions.

It turns out that the first term a_1 dominates the charged lepton masses. For neutrinos a_1 vanishes and the masses are determined to the leading order by the generalized seesaw formula given by the sum term in Eq. (F5).

In the mass formula one can essentially approximate the lepton eigenvector by the zero eigenvector of matrix $\tilde{Y}_0$. The zero eigenvector of matrix $\tilde{Y}_0$ is $\tilde{v}_0 = (1,0,\ldots,0)$. To estimate the accuracy of this approximation and to calculate anomalous couplings to the weak currents one should know the leading-order corrections to vector $\tilde{v}_0$: $\tilde{v}_1 = \tilde{v}_0 + \delta\tilde{v}$. The lepton mass is the smallest eigenvalue of the fermion mass matrix. A standard (numerical) method to find accurate expression for the smallest eigenvector of matrix is to multiply the approximation by inverse of the matrix. It is easily seen that this way the errors of the approximation are reduced at least by factor of m/M, where m is the smallest eigenvalue (physical lepton mass) and M is the second-smallest eigenvalue of the mass matrix (typically the lightest supersymmetric chargino or neutralino).

Thus the leading-order correction to vector $\tilde{v}_0$ is obtained by multiplying it by matrix $\tilde{Y}^{-1}$ and normalizing it.

$$\tilde{v}_1 = \frac{\tilde{Y}^{-1}\tilde{v}_0}{|\tilde{Y}^{-1}\tilde{v}_0|}. \tag{F6}$$

$\tilde{v}_1$ can be calculated to leading order

$$\begin{aligned} (\tilde{Y}^{-1} v_0)_i &= (\tilde{Y}^{-1})_{i1} \\ &= (-1)^{i+1} \frac{|\tilde{Y}_{\hat{i}\hat{1}}|}{|\tilde{Y}|} \\ &\approx \begin{cases} \dfrac{1}{m}, & i = 1, \\ \displaystyle\sum_{j\neq 1} (-1)^{i+j} a_j \frac{|(\hat{Y}_0)_{\widehat{i-1},\widehat{j-1}}|}{m|\hat{Y}_0|}, & i \neq 1. \end{cases} \end{aligned} \tag{F7}$$

The correction δv to eigenvector $\tilde{v}_0$ is thus to the leading order ($\tilde{v}_1=\tilde{v}_0+\delta\tilde{v}$)

$$\delta\tilde{v}_i=\begin{cases} 0, & i=1 \\ \sum_{j\neq 1}(-1)^{i+j}a_j\dfrac{|(\hat{Y}_0)_{\hat{i}-1,\hat{j}-1}|}{|\hat{Y}_0|}, & i\neq 1. \end{cases} \quad \text{(F8)}$$

We need an expression for $\Sigma_{i,i'\neq 1}\delta\tilde{v}_i C_{ii'}\delta\tilde{v}'_{i'}$ to calculate the anomalous coupling to weak currents (see Sec. V B). The dimension of $\delta\tilde{v}_i$ is N and the dimension of $\delta\tilde{v}'_i$ is N'. We can take $N\geqslant N'$ without loss of generality. We further assume that we have permuted the basis so that C is a diagonal matrix with equal diagonal elements grouped together.

Since we want to do algebra with determinants, it is useful to expand some of the matrices to the square form:

$$\hat{Y}''_0=\begin{pmatrix} \hat{Y}'_0 & 0_{N'-1\times N-N'} \\ 0_{N-N'\times N'-1} & 1_{N-N'\times N-N'} \end{pmatrix},$$

$$C'=\begin{pmatrix} \hat{C}_{\hat{1}\hat{1}} & 0_{N'-1\times N-N'} \\ & 1_{N-N'\times N-N'} \end{pmatrix}. \quad \text{(F9)}$$

The required expression is then to leading order

$$\delta\tilde{v}_i C_{ii'}\delta\tilde{v}'_i=\sum_{i,i',j,j'}(-1)^{i+i'+j+j'}a_j a'_{j'}C_{ii'} \times\frac{|(\hat{Y}_0)_{\hat{i}-1,\hat{j}-1}||(\hat{Y}'_0)_{\hat{i}'-1,\hat{j}'-1}|}{|(\hat{Y}_0)||(\hat{Y}'_0)|}$$
$$=\sum_{j,j'}(-1)^{j+j'}a_j a'_{j'}C_{ii}\frac{|(\hat{Y}_0^\dagger\hat{Y}''_0)_{\hat{j}-1,\hat{j}'-1}|}{|\hat{Y}_0^\dagger\hat{Y}''_0|}. \quad \text{(F10)}$$

Due to the properties of the weak current, the matrices appearing in determinants $((\hat{Y}_0^\dagger\hat{Y}''_0)_{\hat{j}-1,\hat{j}'-1}$ and $\hat{Y}_0^\dagger\hat{Y}''_0)$ are block-diagonal matrices with each block corresponding to a constant C_{ii}. The ratios of determinants in Eq. (F10) thus reduces to ratios of these diagonal block matrices.

The general neutral weak current coupling for charginos is of the form

$$\mathcal{L}_{NC}=-\frac{g_L}{\cos\theta_W}Z_L^\mu\bar{\Psi}_i\gamma_\mu\frac{1}{2}(V_i-A_i\gamma^5)\Psi_i$$
$$=-\frac{g_L}{\cos\theta_W}Z_L^\mu\bar{\Psi}_i\gamma_\mu(L_iP_L+R_iP_R)\Psi_i, \quad \text{(F11)}$$

where $P_L=\frac{1}{2}(1-\gamma^5)$ and $P_R=\frac{1}{2}(1+\gamma^5)$.

The chargino mass matrix is of the form (F2). The calculation for Dirac fermions proceeds analogously to the Majorana case discussed above: We define unitary matrices $\tilde{U}_0$ and $\tilde{V}_0$ such that the first row and column of matrix $\tilde{X}_0=\tilde{U}_0^*X_0\tilde{V}_0^\dagger$ vanishes. The a vectors are in this case $a_{Li}=(\tilde{U}_0^*X_1\tilde{V}_0^\dagger)_{1i}$ and $a_{Ri}=(\tilde{U}_0^*X_1\tilde{V}_0^\dagger)_{i1}$.

The correction to the couplings L and R is (as compared to the standard model value)

$$\delta L=\sum_i(I_{3L}^i-I_{3L}^{e_L})v_i^2$$
$$=\sum_{i,j,j'}(-1)^{j+j'}(I_{3L}^i-I_{3L}^{e_L})a_{Lj}a_{Lj'}\frac{|(\hat{X}_0\hat{X}_0^\dagger)_{\hat{j}-1,\hat{j}'-1}|}{|\hat{X}_0\hat{X}_0^\dagger|},$$

$$\delta R=\sum_{i,j,j'}(-1)^{j+j'}(I_{3L}^i-I_{3R}^{e_R})a_{Rj}a_{Rj'}\frac{|(\hat{X}_0^\dagger\hat{X}_0)_{\hat{j}-1,\hat{j}'-1}|}{|\hat{X}_0^\dagger\hat{X}_0|}, \quad \text{(F12)}$$

where I_{3L}^i are the $SU(2)_L$ quantum numbers for the corresponding fields (for lepton interaction eigenstates $I_{3L}^{e_L}=-\frac{1}{2}$ and $I_{3L}^{e_R}=0$).

The correction to axial coupling is $\delta A=\delta L-\delta R$ and to vectorial coupling $\delta V=\delta L+\delta R$.

APPENDIX G: FERMION MASS MATRICES IN MODEL 2B

The chargino vectors and mass matrices in model 2b are

$$\Psi^{+T}=(-i\lambda_L^+,-i\lambda_R^+,\tilde{\phi}_{12}^+,\tilde{\phi}_{22}^+,\tilde{\delta}_R^+,e_R^+),$$

$$\Psi^{-T}=(-i\lambda_L^-,-i\lambda_R^-,\tilde{\phi}_{11}^-,\tilde{\phi}_{21}^-,\tilde{\Delta}_R^-,e_L^-),$$

$$X=\begin{pmatrix} M_L & 0 & 0 & g_Lv_u & 0 & 0 \\ 0 & M_R & -g_Rv_d & 0 & \sqrt{2}g_Rv_{\delta_R} & g_R\sigma_R \\ g_Lv_d & 0 & 2\mu_\phi^{11} & \mu_\phi^{12} & 0 & \lambda_e\sigma_L \\ 0 & -g_Rv_u & \mu_\phi^{21} & 2\mu_\phi^{22} & 0 & \lambda_\nu\sigma_L \\ 0 & -g_Rv_{\Delta_R} & 0 & 0 & \mu_{\Delta_R} & -\sqrt{2}f_R\sigma_R \\ g_L\sigma_L & 0 & -\lambda_e\sigma_R & -\lambda_\nu\sigma_R & 0 & -\lambda_ev_d \end{pmatrix}. \quad \text{(G1)}$$

For neutralinos $\Psi^{0T}=(-i\lambda_L^0,-i\lambda_R^0,-i\lambda_{B-L}^0,\tilde{\phi}_{11}^0,\tilde{\phi}_{12}^0,\tilde{\phi}_{21}^0,\tilde{\phi}_{22}^0,\tilde{\Delta}_R^0,\tilde{\delta}_R^0,\nu_L,\nu_R)$ and

$$Y=\begin{pmatrix} M_L & 0 & 0 & \frac{1}{\sqrt{2}}g_L v_d & 0 & 0 & -\frac{1}{\sqrt{2}}g_L v_d & 0 & 0 & \frac{1}{\sqrt{2}}g_L\sigma_L & 0 \\ & M_R & 0 & -\frac{1}{\sqrt{2}}g_R v_d & 0 & 0 & \frac{1}{\sqrt{2}}g_R v_u & \sqrt{2}g_R v_{\Delta_R} & -\sqrt{2}g_R v_{\delta_R} & 0 & -\frac{1}{\sqrt{2}}g_R\sigma_R \\ & & M_{B-L} & 0 & 0 & 0 & 0 & -\sqrt{2}g_{B-L}v_{\Delta_R} & \sqrt{2}g_{B-L}v_{\delta_R} & -\frac{1}{\sqrt{2}}g_{B-L}\sigma_L & \frac{1}{\sqrt{2}}g_{B-L}\sigma_R \\ & & & 0 & -2\mu_\phi^{11} & 0 & -\mu_\phi^{12} & 0 & 0 & 0 & 0 \\ & & & & 0 & -\mu_\phi^{12} & 0 & 0 & 0 & \lambda_e\sigma_R & \lambda_e\sigma_L \\ & & & & & 0 & -2\mu_\phi^{22} & 0 & 0 & 0 & 0 \\ & & & & & & 0 & 0 & 0 & \lambda_\nu\sigma_R & \lambda_\nu\sigma_L \\ & & & & & & & 0 & \mu_{\Delta R} & 0 & -2f_R\sigma_R \\ & & & & & & & & 0 & 0 & 0 \\ & & & & & & & & & 0 & \lambda_\nu v_u \\ & & & & & & & & & & -2f_R v_{\Delta_R} \end{pmatrix}. \tag{G2}$$

The angles related to the right-handed part of the Dirac spinor are

$$\tan\alpha_R=\sigma_R\frac{\sqrt{g_R^2(\mu_{\Delta R}+2f_R v_{\delta_R})^2+2(f_R M_R-g_R^2 v_{\Delta_R})^2}}{M_R\mu_{\Delta_R}+2g_R^2 v_{\Delta_R}v_{\delta_R}},\quad \tan\alpha_R'=\frac{\sqrt{2}}{g_R}\frac{f_R M_R-g_R^2 v_{\Delta_R}}{\mu_{\Delta R}+2f_R v_{\delta_R}}, \tag{G3}$$

and the angles related to the left-handed part of the Dirac spinor are

$$\tan\alpha_L'=\sigma_R\frac{\sqrt{(\lambda_\nu\mu_\phi^{12}-2\lambda_e\mu_\phi^{22})^2+(\lambda_e\mu_\phi^{12}-2\lambda_\nu\mu_\phi^{11})^2}}{(\mu_\phi^{12})^2-4\mu_\phi^{11}\mu_\phi^{22}},\quad \tan\alpha_L''=\frac{\lambda_e\mu_\phi^{12}-2\lambda_\nu\mu_\phi^{11}}{\lambda_\nu\mu_\phi^{12}-2\lambda_e\mu_\phi^{22}}. \tag{G4}$$

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